MATH NEWS

Grade 8, Module 1, Topic A

LAFAYETTE PARISH SCHOOL SYSTEM

8th Grade Math

Module 1: Integer Exponents and Scientific Notation

Math Parent Letter

This document is created to give parents and students a better understanding of the math concepts found in Eureka Math (© 2013 Common Core, Inc.) that is also posted as the Engage New York material which is taught in the classroom. Module 1 of Eureka Math (Engage New York) builds on exponential notation with integer exponents and transforming expressions in order to perform operations including numbers in scientific notation as well as judging the magnitude of a number.

Focus Area Topic A:

Integer Exponents

Words to Know:

Whole number – the numbers 0,1,2,3,4,5,… on the number line

Integer -the numbers ...,-3,-2,-1,0,1,2,3,... on the number line

Base - the number being raised to a power (or exponent)

Exponent - the number of times a number is to be used as a factor in a multiplication expression

Power- another name for exponent

Squaring a number- multiplying a number by itself; raising a number to the second power

Cubing a number- multiplying a number by itself 3 times; raising a number to the third power

Exponential notation- a method that allows the representation of numbers in shorter form that aids in mathematical calculations. (Example: $8 = 2\times2\times2 = 2^3$ So, 2^3 is considered exponential notation. The base of the exponential notation is 2 and the exponent (or power) is 3

Reciprocal- a multiplicative inverse; the reciprocal of a number is the result when you divide a 1 by that number (example: $\frac{1}{2}$ is the reciprocal of 2 because $1\div2=\frac{1}{2}$)

Expanded form of a decimal- shows how much each digit is worth and that the number is the total of those values added together (Example: 432.56 written in expanded form is $(4\times10^2)+(3\times10^1)+(2\times10^0)+(5\times10^{-1})+(6\times10^{-2})$

Focus Area Topic A:

Integer Exponents

In this module, students build upon the work done with exponents in 5th, 6th, and 7th grades. They will learn the precise definition of exponential notation and how to use this definition to prove the properties of exponents.

3^4 ← exponent; ↑ base — which means $\underbrace{3\times3\times3\times3}_{4 times} = 3^4$

6×6 means 6^2 which is read as "6 squared"

$2\times2\times2$ means 2^3 which is read as "2 cubed"

$$\underbrace{(-5)\times(-5)\times(-5)}_{3 times} = (-5)^3 \qquad \underbrace{\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)}_{2 times} = \left(\frac{2}{3}\right)^2$$

It is important to use parentheses when writing numbers with a negative or fractional base in

A Law of Exponents: *Multiplying*

The laws of exponents are derived from the meaning behind the exponential notation as follows…

$$2^3\times2^4 = \underbrace{\underbrace{2\times2\times2}_{3 times}\times\underbrace{2\times2\times2\times2}_{4 times}}_{7 times}$$

This means that you have 2 times itself (3+4) times, then you can see that $2^3\times2^4=2^{3+4}=2^7$. Using this thinking allows us to build a more abstract equation that works for any base x with positive integers m and n

$$\boldsymbol{x^m\cdot x^n = x^{m+n}} \quad \text{because}$$

$$\boldsymbol{x^m\times x^n = \underbrace{(x\cdots x)}_{m\ times}\times\underbrace{(x\cdots x)}_{n\ times} = \underbrace{(x\cdots x)}_{m+n\ times} = x^{m+n}}$$

Note in these equations, the bases are the same. The next logical question is, "What if the bases aren't the same?"

Unlike Bases

Can be rewritten so bases are the same	Cannot be rewritten so bases are the
$2\cdot4$ $2\cdot2^2$ 2^{1+2} 2^3	$3\cdot2^5$ This cannot be simplified because the bases cannot be the same

Focus Area Topic A:

Integer Exponents

A Law of Exponents: *Division*

Students extend the one property of exponents defined thus far and the definition of exponents to discover a useful consequence. This consequence leads to the Law of Exponents for dividing different powers of a number.

What is $\frac{3^7}{3^5}$?

option #1

$$\frac{3^7}{3^5} = \frac{3\times3\times3\times3\times3\times3\times3}{3\times3\times3\times3\times3}$$

$$\frac{3^7}{3^5} = \frac{3\times3\times3\times3\times3\times3\times3}{3\times3\times3\times3\times3}$$

each $\frac{3}{3}$ is equal to 1

$$\frac{3^7}{3^5} = \frac{3^2}{1} = 3^2$$

by equivalent fractions

$$\frac{3^7}{3^5} = \frac{3^2}{1} = 3^2 = 3^{7-5}$$

option #2

$$\frac{3^7}{3^5} = \frac{3^5\times3^2}{3^5}$$

by $x^m x^n = x^{m+n}$

since $\frac{3^5}{3^5} = 1$

then $\frac{3^7}{3^5} = \frac{3^5\times3^2}{3^5} = \frac{3^2}{1}$

$$\frac{3^7}{3^5} = \frac{3^5\times3^2}{3^5} = \frac{3^2}{1} = 3^{7-5}$$

This leads us to the more abstract conclusion that holds true for any base x and integers m and n. (Take note of the like bases.)

$$\frac{x^m}{x^n} = x^{m-n}$$

When first introduced, we restrict the values of m and n so that $m > n$ until students have a conceptual understanding of negative exponents later in this module topic.

A Law of Exponents: *Powers of a Power*

As we progress through the module, each additional property of exponents is developed using the definition of exponents along with the previously developed properties.

$$(3^4)^5 = (3\text{x}3\text{x}3\text{x}3)^5$$

$$= \underbrace{(3\text{x}3\text{x}3\text{x}3) \text{ x } \dots \text{ x } (3\text{x}3\text{x}3\text{x}3)}_{5 \text{ times}}$$

$$= \underbrace{3\text{x}3\text{x} \dots \text{x}3}_{5\text{x}4 \text{ times}}$$

$$= 3^{5\text{x}4}$$

Like all the other, we can then generalize this for any base x for integers m and n

because $\quad (x^m)^n = x^{mn}$

$$(x^m)^n = (\underbrace{x \cdot x \cdots x}_{m\ times})^n$$

$$= \underbrace{(x \cdot x \cdots x)}_{m\ times} \times \cdots \times \underbrace{(x \cdot x \cdots x)}_{m\ times} \quad (n\ times)$$

$$= x^{mn}$$

This law applies to products of powers raised to powers also.

$$(xy)^n = \underbrace{(xy)\cdots(xy)}_{n\ times}$$

ALERT: Misconception

While this last property seems simple enough, it often is misused. This law works for products **NOT** sums or differences.

We just want you to be aware of the fact...

$$(x+y)^n \neq x^n + y^n \quad \textbf{unless} \quad n = 1$$

example $\quad (2+3)^2 \neq 2^2 + 3^2$

"The Zero-th Power"

Using various methods students derive the definition that for any base x, $x^0 = 1$. One of the methods uses the property for dividing powers with like bases.

$$\frac{x^m}{x^m} = x^{m-m} = x^0 = 1$$

Negative Exponents

Negative exponents are also defined using the previous laws and previous knowledge of reciprocals. $3^5 \cdot 3^{-5} = 3^{5+-5} = 3^0$ Since any base raised to the zero-th power equals 1, then $3^0 = 1$. Let's take a closer look since we know $3^5 \cdot 3^{-5} = 1$. If multiplying 3^5 by the number 3^{-5} has a product of 1 and by definition multiplying a number by its reciprocal has a product of 1, then the number 3^{-5} must be mathematically equal to the reciprocal of 3^5. This reciprocal is $\frac{1}{3^5}$. This idea leads to a formal definition.

For any positive number x and for any positive integer n, we define

$$x^{-n} = \frac{1}{x^n}.$$

Once negative exponents are defined, we can perform operations using this definition and following all of the previous laws.

Example: Simplify $\frac{y^3}{y^{-2}}$

$$\frac{y^3}{y^{-2}} = \frac{y^3}{\frac{1}{y^2}} = y^3 \times \frac{y^2}{1} = y^3 \times y^2 = y^{3+2} = y^5$$

Example Problems and Answers:

Problem: $\frac{\left(\frac{5}{3}\right)^{11}}{\left(\frac{5}{3}\right)^3}$ Answer: $\left(\frac{5}{3}\right)^{11-3} = \left(\frac{5}{3}\right)^8$

Problem: $\frac{4}{m^3}\left(3m^8\right)$ Answer: $\frac{4}{m^3}\left(\frac{3m^8}{1}\right) = \frac{12m^8}{m^3} = 12m^{8-3} = 12m^5$

Problem: $6^3 \cdot 5^2 \cdot 6 \cdot 5^6$ Answer: $6^3 \cdot 5^2 \cdot 6 \cdot 5^6$

$$6^{3+1} \cdot 5^{2+6}$$

$$6^4 \cdot 5^8$$

Throughout this module the emphasis is on understanding the laws of exponents, so encourage your student to write out all steps.