8th Grade Math

Module 3: Similarity

Math Parent Letter

This document is created to give parents and students a better understanding of the math concepts found in Eureka Math (© 2013 Common Core, Inc.) that is also posted as the Engage New York material which is taught in the classroom. Module 3 of Eureka Math (Engage New York) focuses on dilations, similarity, and application of that knowledge to a proof of the Pythagorean Theorem based on the Angle-Angle criterion for similar triangles. The goal of this module is to replace the idea of "same shape, different sizes" with a definition of similarity that can be applied to geometric shapes that are not polygons, such as ellipses and circles. Topic A begins by demonstrating the need for a precise definition of dilation instead of "same shape, different size". Students will begin experimenting with dilations using a compass and straightedge in order to develop conceptual understanding.

Focus Area Topic A: *Dilation*

Words to Know:

Dilation –a rule that moves points in the plane a specific distance, determined by the scale factor (referred to as r), from a center (referred to as O). Dilation in the coordinate plane is a transformation that shrinks or magnifies a figure by multiplying each coordinate of the figure by the scale factor.

Congruence – finite composition of basic rigid motions—reflections, rotations, translations—of the plane. Two figures in a plane are congruent if there is a congruence that maps one figure onto the other figure.

Similar – Two figures in the plane are similar if there exists a similarity transformation taking one figure to the other.

Similarity Transformation – is a composition of a finite number of basic rigid motions or dilations. The scale factor of a similarity transformation is the product of the scale factors of the dilations in the composition; if there are no dilations in the composition the scale factor is defined to be 1.
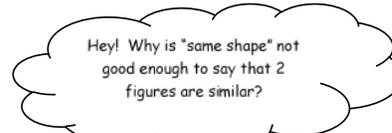
Similarity – an example of a transformation.

EXPLORATORY CHALLENGE: Determine which 2 characters are similar.

Solution: B and C
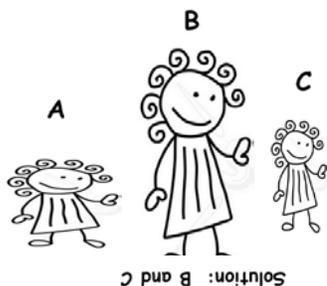
Focus Area Topic A: *Dilation*

What Lies Behind "Same Shape"?

Hey! Why is "same shape" not good enough to say that 2 figures are similar?

In this lesson, dilation is defined and the role of scale factor is demonstrated through the shrinking and magnification of figures. "Same shape" is not good enough because the shapes must be proportional.

What exactly is a dilation? It is a rule that moves points in the plane a specific distance, determined by the scale factor (designated with the variable r) from a center O.

- To magnify a figure, each of the sides that comprise the figure are multiplied by a scale factor, r, which is greater than one. This would be where $r > 1$.

> In the **EXPLORATORY CHALLENGE**, each side on **Figure C** was magnified by a scale factor of 2 to create **Figure B.** In this drawing, **Figure C** stands 0.7 inches tall and 0.35 inches wide; therefore, **Figure B** stands 1.4 inches tall and 0.7 inches wide.
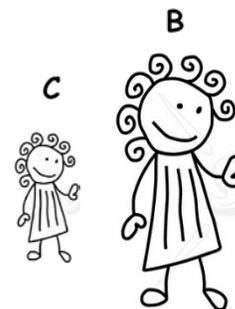
- To shrink a figure, all of the points that comprise the figure are multiplied by a scale factor, r, which is between zero and one. This would be where $0 < r < 1$.

Use the same **Figure B** above to create **Figure C.** All of the lengths that comprise **Figure B** are multiplied by a scale factor of ½…so that the original height of 1.4 inches becomes 0.7 inches and the width of 0.7 inches becomes 0.35 inches.

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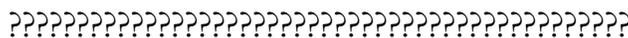
What scale factor would produce an exact copy, a congruent figure?

SCALE FACTOR of 1 produces CONGRUENT FIGURES.

Students discover that dilations map lines to lines, rays to rays, and segments to segments by simply dilating two points on the line, ray, or segment. Dilations also map angles to angles of the same degree. Now that we have established some properties of dilations, let's discuss how to perform dilations.

Given center O and triangle ABC, let's dilate the triangle from center O with a scale factor $r = 3$. Since the scale factor is a whole number, we really have two methods from which to choose. One method is using a straightedge and compass, and the second would be with a ruler.
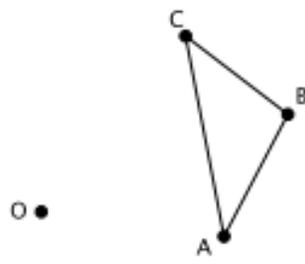
Step 1: Both of these methods rely on the fact that the center of dilation, the original point, and the dilated point are all collinear (meaning they all lie on the same line). Both methods begin by drawing rays originating from the center of dilation, O, extending through each vertex.
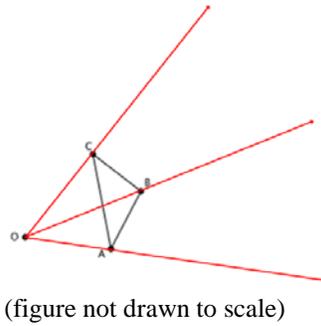
(figure not drawn to scale)

Step 2: The next step depends on the tools that are selected. With a straightedge and compass, students would set the point of the compass at the center of dilation, O. The compass should then be opened until the pencil reaches one of the vertices. This sets the distance between those two points. Without changing the opening, move the point from center O to the actual vertex on the figure; construct an arc that intersects the ray. The point of intersection of the arc and the ray would be the location on the new dilated point if the scale factor was $r = 2$.

Since we are tripling the size of the figure ($r = 3$), we would then repeat these steps one more time. We would repeat these steps for each vertex. Once all vertices are dilated, we would connect the new vertices. This would be the end result.
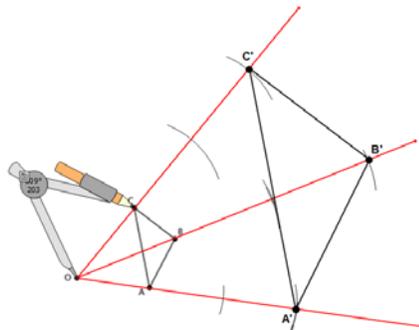
With a ruler, students would measure from the center O, to each vertex. Each of these measurements will then be multiplied by the scale factor, in this case by 3.

This is one example. Using this example, students will measure angles to show the angles are congruent.
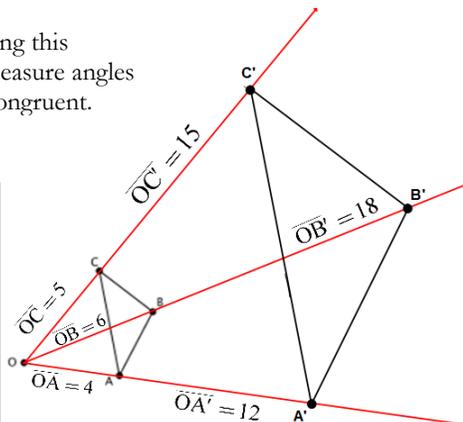
Make the connection:
The scale factor, r, can be calculated by the ratio $\frac{new\ length}{original\ length}$.

In this example,
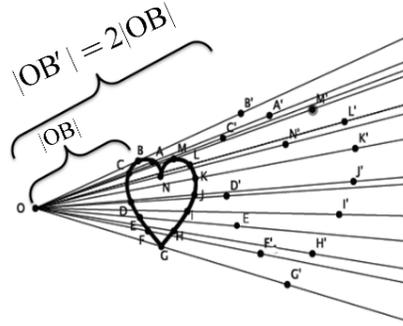$$r = \frac{|OC'|}{|OC|} = \frac{15}{5} = 3$$

Examples of Dilations

The following are example is from the Problem Set in Lesson 3.

- Can you dilate the heart from center O by a scale factor r = 2?
- Approximately how many points will be needed to make a good image of the original?
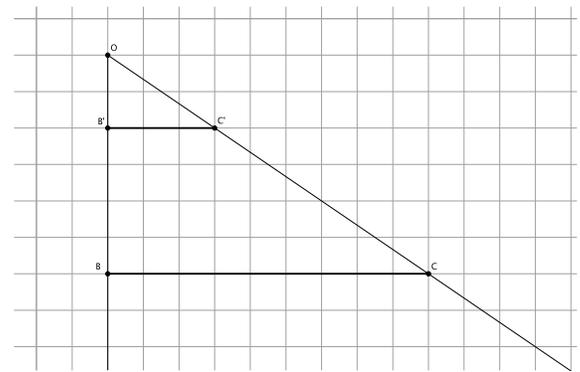
♥♥♥♥♥POSSIBLE SOLUTION♥♥♥♥♥

$|OB'| = 2|OB|$

$|OB|$

Notation Alert:

$$|OB|$$

in the context of geometry means "the length of segment OB."

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FTS: FUNDAMENTAL THEOREM OF SIMILARITY

Caleb sketched the diagram above on graph paper. He dilated points B and C from center O.
- What is the scale factor?
- Which segments are parallel and how do you know?
- Which angles are equal in measure and how do you know?

Solution

- r = ⅓
- Segments BC and B'C' are parallel: they lie on the lines of graph paper, which are parallel.
- $\angle OB'C' \cong \angle OBC$; $\angle OC'B' \cong \angle OCB$—they are corresponding angles of parallel lines cut by a transversal

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The Fundamental Theorem of Similarity (FTS) is used to develop the rules for dilating points in the coordinate plane. Students apply the Fundamental Theorem of Similarity to determine that if the center of dilation is the origin (ordered pair (0, 0)) then any point (x, y) can be dilated by a scale factor of r and represented by (rx, ry). The foundation is set to develop the rules of Similar Figures in Module 3 Topic B.