# MATH NEWS

Grade 8, Module 3, Topic B

# 8th Grade Math

*Module 3: Similarity*

**Math Parent Letter**

This document is created to give parents and students a better understanding of the math concepts found in Eureka Math (© 2013 Common Core, Inc.) that is also posted as the Engage New York material which is taught in the classroom. Module 3 of Eureka Math (Engage New York) focuses on dilations, similarity, and application of that knowledge to a proof of the Pythagorean Theorem based on the Angle-Angle criterion for similar triangles. The goal of this module is to replace the idea of "same shape, different sizes" with a definition of similarity that can be applied to geometric shapes that are not polygons, such as ellipses and circles. Topic B begins with the definition of similarity and the properties of similarities.

## Focus Area Topic B: *Similar Figures*

**Words to Know:**

**Dilation** –a rule that moves points in the plane a specific distance, determined by the scale factor (referred to as *r*), from a center (referred to as *O*). Dilation in the coordinate plane is a transformation that shrinks or magnifies a figure by multiplying each coordinate of the figure by the scale factor. (Remember that a scale factor is a number that tells you how many times bigger one length is than another. Ex: A scale factor of 3 means that the new length is 3 times that of the original. A scale factor of $\frac{1}{3}$ tells you that the new length is $\frac{1}{3}$ the length of the original.)

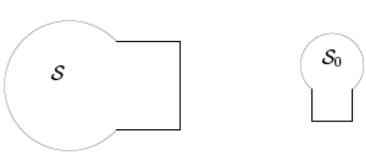
**Congruence** – finite composition of basic rigid motions—reflections, rotations, translations—of the plane. Two figures in a plane are congruent if there is a congruence that maps one figure onto the other figure.

**Similar** – Two figures in the plane are similar if there exists a similarity transformation taking one figure to the other. (symbol ~)

**Similarity Transformation** – is a composition of a finite number of basic rigid motions or dilations. The scale factor of a similarity transformation is the product of the scale factors of the dilations in the composition; if there are no dilations in the composition the scale factor is defined to be 1.

**Similarity** – an example of a transformation.

Is a dilation alone enough to state that 2 figures below are similar? Do the following figures look similar?



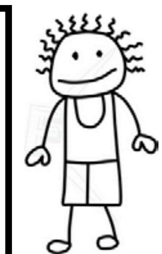

**Notation Alert:** $S_0$ can be read as "S sub zero" where sub is short for subscript, but it is most often read "S naught." Both of which often denote "the original" figure or value.

In order for these two figures to be truly similar there MUST exist a dilation (meaning the two figures are in proportion) followed by a congruence (a sequence of basic rigid motions).

## Focus Area Topic B:

*Similar Figures*

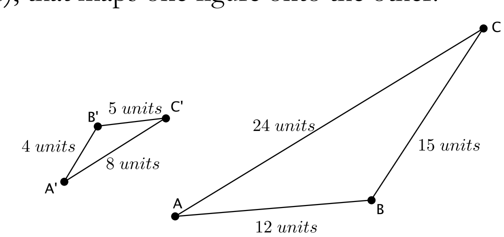
- What do we need to do to prove that the 2 figures below are similar?
- Can we dilate 1 figure so that it is the same size as the other?
- Is a dilation alone enough proof? If not, what else do we need to do?



- To prove that they're similar, we need to show that you can dilate one to make the other.
- We'd need an appropriate scale factor.
- Dilating alone wouldn't prove similarity. We'd also need to translate and rotate until S maps onto $S_0$.

**Similarity**

To show that a figure in the plane is similar to another figure of a different size, you must describe the sequence of a dilation followed by a congruence (one or more rigid motions), that maps one figure onto the other.



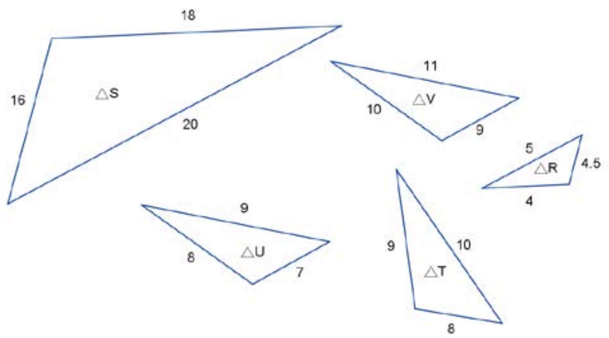

Describe the sequence that would map △A'B'C' onto △ABC and prove that the two triangles are similar.

**SOLUTION:**

- Dilate A'B'C' from point O by a scale factor r = 3. (A'C' is 8 units long so 8 x 3= 24 the length of AC; A'B' is 4 units long so 4 x 3 = 12 the length of AB; B'C' is 5 units long so 5 x 3 =15 the length of BC)
- Then, translate along vector A'A.
- Finally, reflect across line AC.

## Basic Properties of Similarity

The goal of the following exercise is to help students to develop an intuitive sense that similarity is a symmetric and transitive relation. (Symmetric meaning that if one figure is similar to another, $S \sim S'$, then we can be certain that $S' \sim S$. Transitive meaning that given two similar figures, $S \sim T$, and another pair of similar figures, $T \sim U$, then we know that $S \sim U$.)




- Which triangles, if any, have similarity that is symmetric?
- Which triangles, if any have similarity that is transitive?

The first criteria to show similarity uses the ratio for the scale factor $r = \frac{new\ length}{original\ length}$ to compare ratios of corresponding sides. Since no natural correspondence exists, begin by comparing the two longest side lengths, the two shortest side lengths, and the remaining two side lengths. Since we don't really know which one is the original triangle and which is the new triangle, the order doesn't really matter until we need to interpret the scale factor. Here is one example: Consider △S and △R. $r = \frac{20}{5} = \frac{16}{4} = \frac{18}{4.5} = 4$ In this example, we are considering △S as the new triangle (side lengths are 20, 16, and 18 units), so the scale factor tells us that the side lengths of the new triangle are 4 times that of the original. If we consider △R as the new triangle then we would have scale factor $r = \frac{5}{20} = \frac{4}{16} = \frac{4.5}{18} = \frac{1}{4}$. This tells us the side lengths of △R are $\frac{1}{4}$ the side lengths of △S. Since we are only trying to determine similarity, the value of the scale factor isn't the most important fact. The most important fact is that the value of each ratio is equal. Once we have determined that a dilation exists, a congruence could then be determined. Now we determine that the triangles are indeed similar.

**SOLUTION:** The following triangles have similarity that is symmetric:

△S ~ △R and △R ~ △S

△S ~ △T and △T ~ △S

△T ~ △R and △R ~ △T

> **Notation Alert:** △S ~ △R is read "triangle S is similar to triangle R."

The following triangles have similarity that is transitive:

Since △S ~ △R and △R ~ △T, then △S ~ △T.

**Solution** to "Modeling Using Similarity" problem on **bottom right column.**

The triangles are similar by AA criterion.
Distance from the mirror to the building : 1750 - 7.2 = 1742.8
Proportionally: $\frac{x}{5.3} = \frac{1742.8}{7.2}$

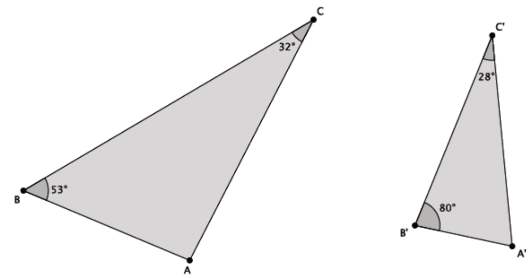
$$5.3\left(\frac{x}{5.3}\right) = 5.3\left(\frac{1742.8}{7.2}\right)$$

$$x = 1282.9 \text{ feet}$$

## Informal Proof of AA (Angle Angle) Criterion for Similarity

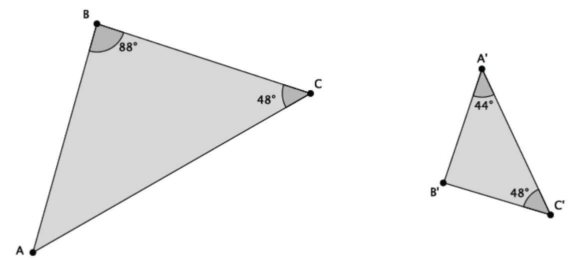
Are the triangles below similar? Can you make an informal argument as to why they are or why they are not?

Example 1




Are the triangles below similar? Can you make an informal argument as to why they are or why they are not?

Example 2

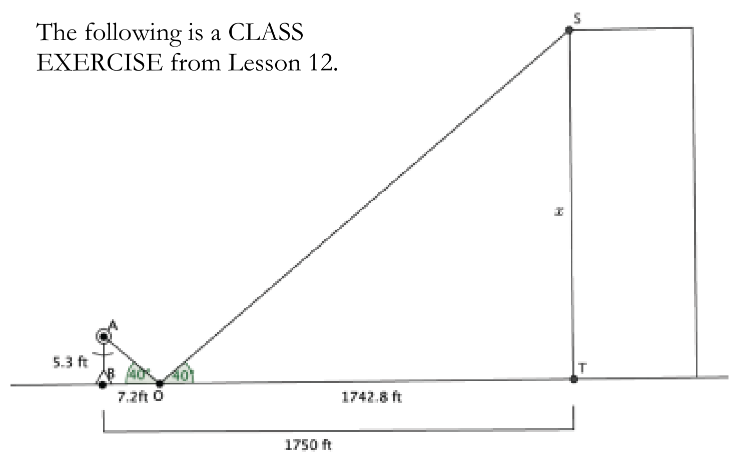



Students are engaged in a discussion that proves two triangles are similar if they have two pairs of corresponding angles that are equal. (Commonly referred to as Angle Angle Criterion for Similarity)

- The triangles in EXAMPLE 1 are **not similar**. Similar figures have angles that are congruent and there are no congruent angles.
- The triangles in EXAMPLE 2 are **similar**. These two triangles have two pairs of congruent angles.

## Modeling Using Similarity

The following is a CLASS EXERCISE from Lesson 12.




You want to determine the approximate height of the building. You place a mirror some distance from yourself (marked as O above) so that you can see the top of the building in the mirror, then you can indirectly measure the height using similar triangles. The distance from eye-level to the ground is 5.3 feet. The distance from you to the mirror is 7.2 feet. The distance from the figure to the base of the building is 1,750 feet. The height of the building will be represented by $x$. So, what is the height of the building?

Note: The marked angles are congruent because the angle of incidence is equal to the angle of reflection. Also, it is understood that you and the building are perpendicular to the ground.

(See answer of bottom of the left column)