MATH NEWS
Grade 8, Module 3, Topic C

8th Grade Math

Module 3: Similarity

Math Parent Letter

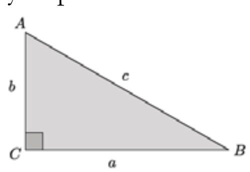
This document is created to give parents and students a better understanding of the math concepts found in Eureka Math (© 2013 Common Core, Inc.) that is also posted as the Engage New York material which is taught in the classroom. Module 3 of Eureka Math (Engage New York) focuses on dilations, similarity, and application of that knowledge to a proof of the Pythagorean Theorem based on the Angle-Angle criterion for similar triangles.

Focus Area Topic C:

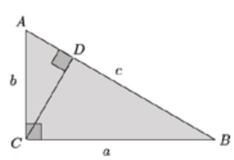
Pythagorean Theorem

The following proof of the Pythagorean Theorem is based on the fact that similarity is transitive. (Transitive meaning that given two similar figures, S~T, and another pair of similar figures, T~U, then we know that S~U.)

Let's begin with a right triangle, △ABC, and use what we know about similarity to prove that $a^2 + b^2 = c^2$.



Now we are going to split △ABC into two other right triangles by drawing a line segment from vertex C to point D so that the line is perpendicular to side AB…like this…



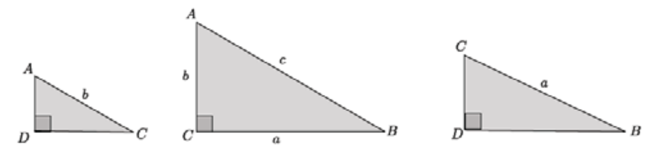
This line segment, $\overline{CD}$, will divide the original triangle into three similar triangles. Can you name the 3 triangles?

Solution: ΔABC (original), ΔACD, ΔBCD

Next, let's change the layout of the triangles so that we can see why the three are similar. Using our basic rigid motions, we'll separate the three triangles. This will ensure that the lengths of the segments and degrees of the angles are preserved.

Focus Area Topic C:

Pythagorean Theorem



In order to have similar triangles, we must have two common angles according to the AA (Angle Angle) criterion.

Which angles prove that ΔADC and ΔACB are similar?
Answer: *They both have right angles, and they share* ∠A.

What must that mean about ∠C from ΔADC and ∠B from ΔACB?
Answer: *It means that the angles correspond and must be equal because of the triangle sum theorem.*

Which angles prove that ΔACB and ΔCDB are similar?
Answer: *They both have right angles, and they share* ∠B .

What must that mean about ∠A from ΔACB and ∠C from ΔCDB?
Answer: *It means that the angles correspond and must be equal because of the triangle sum theorem.*

If △ADC is similar to △ACB and △ACB is similar to △CDB, is it true that △ADC is similar to △CDB?... How do you know?

Answer: *Yes! The triangles are similar because similarity is a transitive relation.*

OK…we know that the side lengths of similar triangles are proportional. So, if we consider △ADC and △ACB, we can write…

$$\frac{|AC|}{|AB|} = \frac{|AD|}{|AC|}$$

Then by using properties of equality, we get…

$$|AB||AC| \cdot \frac{|AC|}{|AB|} = \frac{|AD|}{|AC|} \cdot |AB||AC|$$

$$\cancel{|AB|}|AC| \cdot \frac{|AC|}{\cancel{|AB|}} = \frac{|AD|}{\cancel{|AC|}} \cdot |AB|\cancel{|AC|}$$

$$|AC|^2 = |AB| \cdot |AD|$$

Now, considering $\triangle ACB$ and $\triangle CDB$, we can write…

$$\frac{|BA|}{|BC|} = \frac{|BC|}{|BD|}.$$

Then using properties of equality again, we get…

$$|BC|^2 = |BA| \cdot |BD|.$$

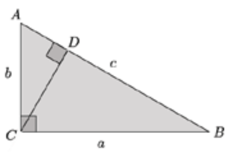
Let's add the two equations together and we have…

$$|AC|^2 + |BC|^2 = |AB| \cdot |AD| + |BA| \cdot |BD|.$$

Using the distributive property, we can rewrite the right side of the equation because there is a common factor $|AB|$ and this gives us…

$$|AC|^2 + |BC|^2 = |AB|(|AD| + |BD|).$$

WOW! We are almost there! Remember our goal: prove that $a^2 + b^2 = c^2$. Let's use the diagram where all three triangles are within one.



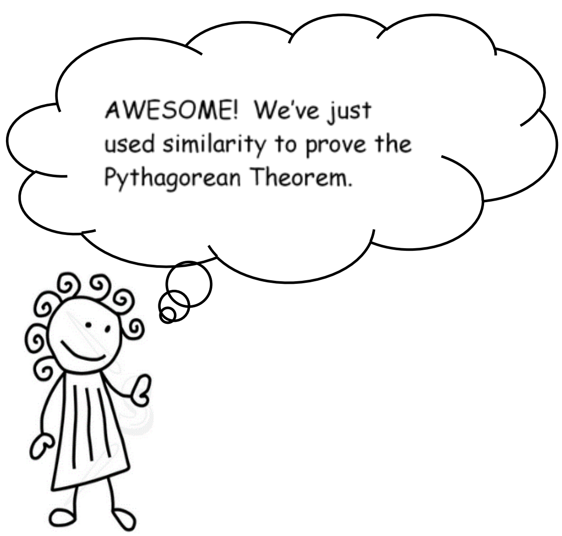
What side lengths are represented by $|AC|^2 + |BC|^2$?

Answer: AC *is side length b, and* BC *is side length a, so the left side of our equation represents* $a^2 + b^2$.

The right side of our equation is $|AB|(|AD| + |BD|)$. Remember, we want this to be equal to c^2. Is it c^2?

YES! If we add the lengths AD and BD we get the entire length of AB and this gives us

$$|AB|(|AD| + |BD|) = |AB| \cdot |AB| = |AB|^2 = c^2.$$



http://www.youtube.com/watch?v=QCyvxYLFSfU

CONVERSE of the PYTHAGOREAN THEOREM

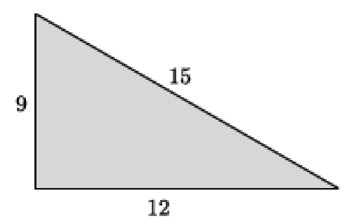
In Modules 2 and 3, two different proofs of the Pythagorean Theorem have been presented:

If the lengths of the legs of a right triangle are a *and* b*, and the length of the hypotenuse is c, then* $\mathbf{a^2 + b^2 = c^2}$.

The **theorem** has a **converse**:

If the lengths of three sides of a triangle, a, b, c, *satisfy* $\mathbf{c^2 = a^2 + b^2}$, *then the triangle is a right triangle, and furthermore, the side of length* c *is opposite the right angle*

The following is an example from Lesson 14's Class Exercises:



The numbers in the diagram indicate the units of lengths of each side of the triangle. Is the triangle shown a right triangle?

SOLUTION: Yes.

$9^2 + 12^2 = 15^2$

$81 + 144 = 225$

$225 = 225$