# MATH NEWS

Grade 8, Module 5, Topic A

## 8th Grade Math

*Module 5: Examples of Functions from Geometry*

**Math Parent Letter**

This document is created to give parents and students a better understanding of the math concepts found in Eureka Math (© 2013 Common Core, Inc.) that is also posted as the Engage New York material which is taught in the classroom. Module 5 of Eureka Math (Engage New York) introduces functions in the context of linear equations along with area and volume formulas. Students will define, evaluate, and compare linear functions using equations of lines and non-linear functions using area and volume formulas.

### Focus Area Topic A: *Functions*

**Words to Know:**

**function –** a rule that assigns to each input exactly one output. Since the rule assigns exactly one output to each input, functions are used to make predictions about real life situations.

**input-** the number or piece of data that is put into a function.

**output-** the number or piece of data that is the result of an input of a function.

**average rate of speed-** total distance traveled divided by the total time elapsed. Average rate of speed is typically calculated when the rate of speed is not constant.

$$average\ rate\ of\ speed = \frac{distance\ traveled\ over\ a\ given\ time\ interval}{time\ interval}$$

**Constant Speeds vs. Average Speeds**

Constant speeds allow us to make predictions with accuracy about the distance traveled in a certain time interval or predictions about the amount of time required to travel a certain distance. For example, if an object travels 256 feet in 4 seconds at a **constant speed** then we can accurately predict that the object would travel half as far in half the amount of time, 128 feet in 2 seconds. Again, continuing the same pattern brings us to the **unit rate** of 64 feet per second since the object can travel 64 feet in 1 second. Because this was identified as a constant speed we can then write a linear equation $y = 64x$ where $y$ represents the distance traveled and $x$ represents the number of seconds.

| Number of seconds $(x)$ | Distance traveled in feet $(y)$ |
|---|---|
| 1 | 64 |
| 2 | 128 |
| 3 | 192 |
| 4 | 256 |

### Focus Area Topic A: *Functions*

**Constant Speeds vs. Average Speeds continued**

If we are not told that there is a constant rate of speed, then at best we can only think in terms of an average rate of speed. Consider this table of data.

| Number of seconds $(x)$ | Distance traveled in feet $(y)$ |
|---|---|
| 0.5 | 4 |
| 1 | 16 |
| 1.5 | 36 |
| 2 | 64 |
| 2.5 | 100 |
| 3 | 144 |
| 3.5 | 196 |
| 4 | 256 |

If you notice, between 0.5 seconds and 1 second the object traveled 12 feet (from 4 feet to 16 feet). Between 1 second and 1.5 seconds, the object traveled 20 feet (36-16=20). Likewise, between 1.5 seconds and 2 seconds the object traveled 28 feet. Each of these time intervals is 0.5 seconds. What we can then conclude is that the object is not traveling the same distance on each 0.5 seconds time interval. This would **NOT** be considered a constant speed. At best, we can discuss average rate of speeds for any of these time intervals.

*Example*: Average rate speed between 0.5 seconds and 1 second

$$average\ rate\ of\ speed = \frac{distance\ traveled\ over\ a\ given\ time\ interval}{time\ interval}$$

$$\frac{distance\ traveled\ over\ a\ given\ time\ interval}{time\ interval} = \frac{16-4}{1-0.5} = \frac{12}{0.5} = 24$$

So, the average rate of speed between 0.5 seconds and 1 second is 24 feet per second. But between 1 second and 1.5 seconds the average rate of speed would be…

$$\frac{distance\ traveled\ over\ a\ given\ time\ interval}{time\ interval} = \frac{36-16}{1.5-1} = \frac{20}{0.5} = 40$$

It appears that the object is actually accelerating, or getting faster since this average rate of speed is 40 feet per second. Now, we could calculate the average rate of speed for the object from 0.5 seconds to 4 seconds.

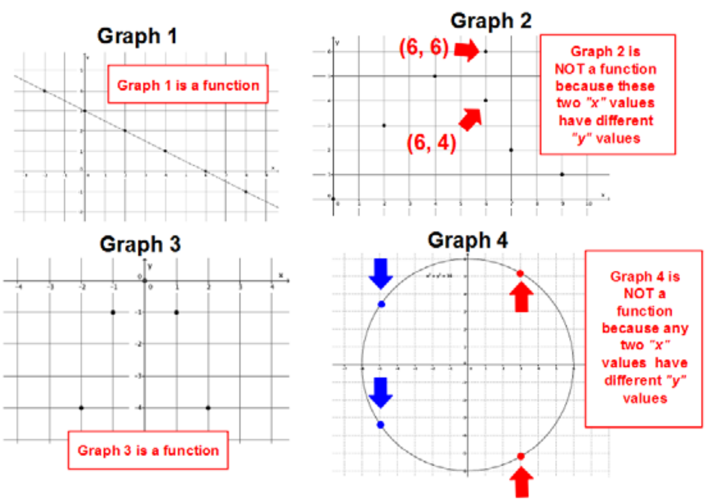
$$\frac{distance\ traveled\ over\ a\ given\ time\ interval}{time\ interval} = \frac{256-4}{4-0.5} = \frac{252}{3.5} \approx 73.1$$

So the object traveled an average rate of speed of 73.1 feet per second between 0.5 seconds and 4 seconds represented in the table. As you saw in the table, the object wasn't exactly traveling 73.1 feet per second for each second it traveled. This was simply the average rate of speed on the 3.5 second interval represented in the table.

**So what is a "function?"**

A function is defined as a rule that assigns to each input exactly one output. Since the rule assigns exactly one output to each input, functions are used to make predictions about real life situations. Functions can be represented as a table, a graph, a rule, an equation or formula, or as a verbal description describing the relationship. Functions are often compared to a machine. Think about a machine that allows you to put something in, and you are guaranteed exactly one output. With this "function machine" you can make accurate predictions because the machine will produce **only** one output. This link is for an interactive demonstration of this idea of a function as a machine generating a single output for each input.
http://www.mathplayground.com/functionmachine.html

**Function: What it is and what it's not**




**Discrete Rates vs. Continuous Rates**

Functions whose rates can only have integer inputs (basically, no fractional pieces) in order to make sense are said to have **discrete rates**. An example of a discrete rate may be the cost per book when you purchase books. It will only make sense for you to purchase a whole number of books. You would not be allowed to purchase half of a book.
Those functions whose rates do allow for fractional input values are considered **continuous rates**. An example of a continuous rate function could be the function that calculates the distance a car travels after a certain time interval since we can input fractions of an hour or even fractions of a minute depending on the units used in the rate.

**Linear vs. Non-linear Functions**

When given a table or graph, sometimes we can determine the rule or formula for a function and sometimes we can't. In the cases where we can't, sometimes it is because we simply haven't been exposed to certain non-linear functions yet. As students progress through their high school math courses, they will learn more about those equations.

**Linear vs. Non-linear Functions continued**

For now students are only expected to determine if a function is linear or non-linear. Grade 8 students have the skills required to write a formula or an equation for linear functions based on the work in Module 4.

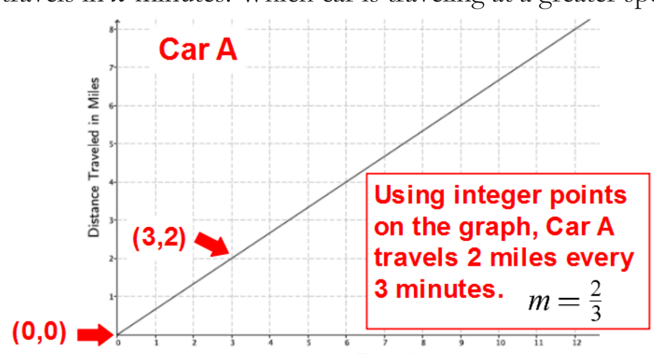
*Example:* The distance Scott walks is a function of the time he spends walking. Scott can walk $\frac{1}{2}$ mile every 8 minutes. Assume he walks at a constant rate. Write an equation to represents the function for the distance that Scott can walk, $y$, in $x$ minutes.
*Solution:* Since this is a constant rate and it is proportional (See Module 4 Topic B Newsletter), then we can establish

$$\frac{0.5}{8} = \frac{y}{x}$$
$$x \cdot \frac{0.5}{8} = \frac{y}{x} \cdot x$$
$$x \cdot \frac{0.5}{8} = \frac{y}{\cancel{x}} \cdot \cancel{x}$$
$$x \cdot \frac{1}{16} = y$$
$$y = \frac{1}{16}x$$

**Note**: This constant rate is also a continuous rate because we can consider distances for fractions of a minute.

Then using this equation, determine how many miles Scott can walk in 24 minutes.
Let $x = 24$
$$y = \frac{1}{16}x = \frac{1}{16}(24) = 1.5$$
So, Scott can walk 1.5 miles in 24 minutes.
Since this constant rate is also a continuous rate, graphing the equation for this function $y = \frac{1}{16}x$ would result in a line because the fractional data points that lie on that line will make sense in this situation.

*Example:* The graph below represents the distance, $y$, Car A travels in $x$ minutes. The table represents the distance, $y$, Car B travels in $x$ minutes. Which car is traveling at a greater speed?




**Car B**

| Time in minutes ($x$) | Distance ($y$) |
|---|---|
| 15 | 12.5 |
| 30 | 25 |
| 45 | 37.5 |

**From the table, the rate the car is traveling is constant because**

$$\frac{25-12.5}{30-15} = \frac{12.5}{15} = \frac{25}{30} = \frac{5}{6}$$
$$\frac{37.5-25}{45-30} = \frac{12.5}{15} = \frac{5}{6}$$
$$\frac{37.5-12.5}{45-15} = \frac{25}{30} = \frac{5}{6}$$

Because $\frac{5}{6}$ is greater than $\frac{2}{3}$ (since $\frac{2}{3} = \frac{4}{6}$) then Car B is traveling at a greater speed.