MATH NEWS
Grade 8, Module 5, Topic B
LAFAYETTE PARISH SCHOOL SYSTEM

8th Grade Math

Module 5: Examples of Functions from Geometry

Math Parent Letter

This document is created to give parents and students a better understanding of the math concepts found in Eureka Math (© 2013 Common Core, Inc.) that is also posted as the Engage New York material which is taught in the classroom. Module 5 of Eureka Math (Engage New York) introduces functions in the context of linear equations along with area and volume formulas. Students will define, evaluate, and compare linear functions using equations of lines and non-linear functions using area and volume formulas.

Focus Area Topic B: *Volume*

Words to Know:

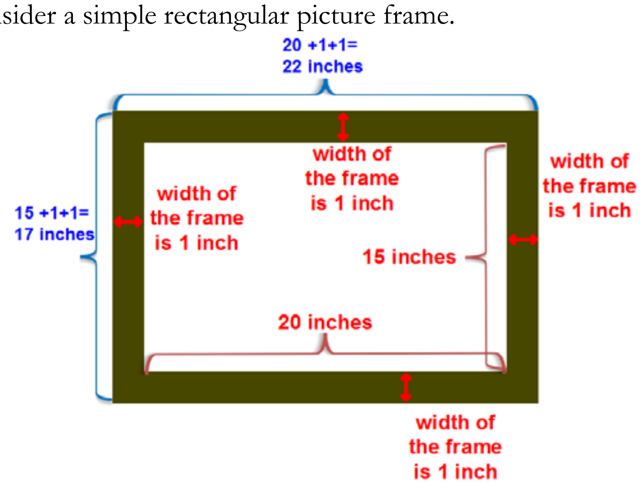
volume – the space that a three-dimensional figure can occupy. For example, the volume of a glass is the amount of liquid it can hold. (from grade 6)

area- the number of square units that covers a two dimensional figure (from grade 5)

solid- a three dimensional geometric figure

Area of a Picture Frame

Area of two dimensional figures is not new to 8th grade, but in 8th grade students explore area of more complex shapes. Let's consider a simple rectangular picture frame.



One way of thinking of this is to decompose this figure into a large rectangle (the outer edge of the frame) and a smaller inner rectangle (the portion where the picture would fit). If we calculate the area of the larger outer rectangle then subtract the area of the smaller inner rectangle, that should leave only the area of the border that makes the picture frame.

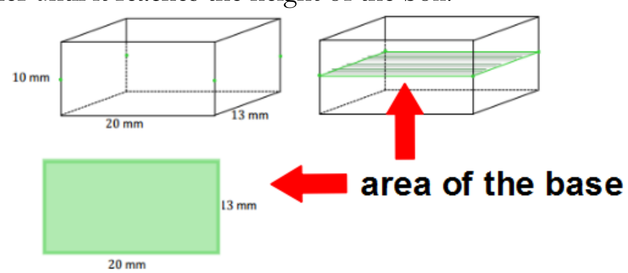
$$area\ of\ frame = (22)(17) - (20)(15) = 374 - 300 = 74$$

So, the area of the frame is 74 square inches.

Focus Area Topic B: *Volume*

Beyond a Formula

So, how do we think about volume? In 5th and 6th grades, students learn how to calculate the volume of rectangular prisms. You can think of rectangular prisms as nothing more than a rectangular box, like a Kleenex® box, a cereal box, or a packing box. They begin by thinking about the base or "bottom" of the box. After finding the area of this base, students can then think about stacking copies or duplicates of that base as thin layers perfectly lined up one on top of the other until it reaches the height of the box.



If these areas are duplicated and stacked in very thin layers it would take the total height of the box to fill it to the top. In this example the volume would be the area of the base 260 square mm $(20 \cdot 13 = 260)$ multiplied by the height of 10 mm. So the volume would be $260 \cdot 10 = 2600$, so 2,600 cubic mm.

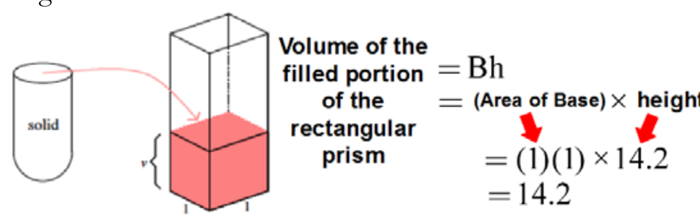
As with most concepts in mathematics, we transition from a concrete example to a more abstract situation. Students will then generalize this idea of volume as the area of the base multiplied by height to geometric solids other than just the rectangular prism.

$$Volume = (Area\ of\ Base) \times (Height)$$

$Volume = Bh$ where B represents the Area of the Base and h represents the height.

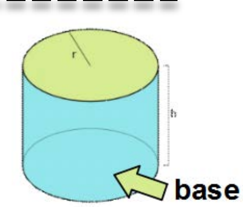
More Complex Problems Using Volume

Example: The solid shown was filled with water and then poured into the standard rectangular prim as shown. The height reaches is 14.2 in. What is the volume of the solid?



Volume of Cylinders

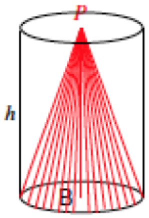
Just like volume of rectangular prisms, volume of cylinders can be thought of as the area of the base multiplied by the height. The base of cylinder is a circle.



$$Volume\ of\ cylinder = Bh = (\pi r^2)h$$

Volume of Cones

The volume of a cone begins with the concept of a cylinder of the same height. Like the cylinder, the cone's base is also a circle. Consider a cone whose base is the same size circle as a cylinder and the heights of the cone and cylinder are the same. You would end up with something like this.



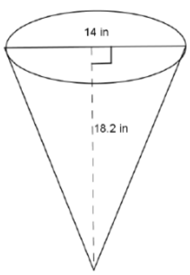
Since the cone fits inside the cylinder, it is safe to say that the volume of the cone should be some fraction of the volume of the cylinder. So, how many cones can fit into the cylinder? Watch the video demonstration.

https://youtu.be/0ZACAU4SGyM

If you watched, you know that three cones can fit inside the cylinder provided they have the same size base and same height.

$$Volume\ of\ Cone = \frac{1}{3}(\pi r^2)h$$

Example: Use the diagram to find the volume of the cone.



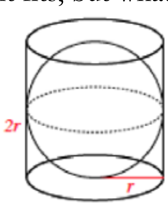
The diameter is 14 inches, so the radius is 7 inches.

$$\textbf{Volume of Cone} = \frac{1}{3}\left(\pi r^2\right)h$$
$$= \frac{1}{3}(\pi)\left(7^2\right)18.2$$
$$= \frac{1}{3}(\pi)(49)18.2$$
$$\approx 297.27\pi$$
$$\approx 933.42$$

So, the volume of the cone is approximately 933.42 cubic inches.

Volume of Spheres

While a two dimensional circle is the set of all points that are the same distance (we call this distance the radius) from a point (we call the center of the circle), a **sphere** is really the set of all points in a three-dimensional space that are the same distance (we call this distance the radius) from a point (we call the center). In an attempt to simplify, a sphere is a 3-D version of a circle. A ball is an example of a sphere. When we discuss volume of a sphere, we really are talking about the volume inside the surface of the sphere. Since a sphere doesn't really have a base the way that cylinders and cones did, thinking about volume is a bit more difficult than the area of the base multiplied by the height. We can, however, think about a sphere fitting inside a cylinder of the same diameter and height. In other words, think of putting the sphere inside a cylinder in which it fits perfectly snug. So, again we know that this sphere should be some fraction of the volume of the cylinder in which it fits, but what fraction?

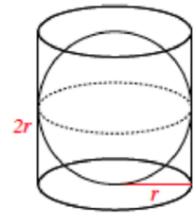


This video allows us to determine the value of that fraction.

https://youtu.be/aLyQddyY8ik

$$Volume\ of\ Sphere = \frac{2}{3}(volume\ of\ cylinder\ with\ the\ same\ diameter\ and\ height)$$

Volume of Spheres continued



Volume of this cylinder $= \mathrm{Bh}$

$= \textbf{(Area of Base)} \times \textbf{height}$

$$= \left(\pi r^2\right)(2r)$$
$$= 2\pi r^3$$

So, $\frac{2}{3}$ of the volume of this cylinder would be...

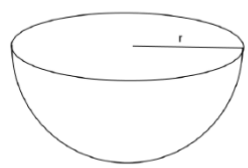
$$\frac{2}{3}\left(2\pi r^3\right) = \frac{4}{3}\pi r^3$$

Which leads us to the formula:

$$Volume\ of\ sphere = \frac{4}{3}\pi r^3$$

Hemisphere

A hemisphere is simply half of a sphere. There is no need to try to remember a formula for volume of a hemisphere. It is simply half of the volume of a sphere.

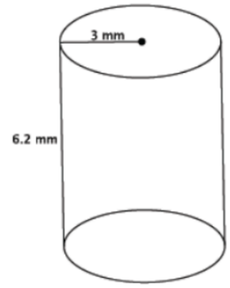


Volume for hemisphere $= \frac{1}{2}$(Volume of sphere)

$$= \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$
$$= \frac{2}{3}\pi r^3$$

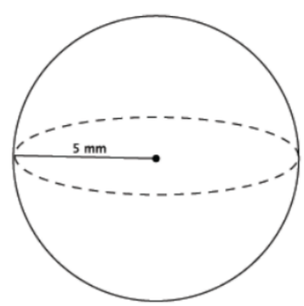
Comparing Volumes

Example: Which has the greater volume?



Volume of cylinder

$$= \mathrm{Bh}$$
$$= \pi r^2 h$$
$$= \pi\left(3^2\right)(6.2) = \pi(55.8)$$



Volume of sphere

$$= \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi\left(5^3\right) = \frac{4}{3}\pi(125)$$
$$= \frac{500}{3}\pi = 166\frac{2}{3}\pi$$

Answer:

So, the sphere has the greater volume.