Focus Area  Topic C:  
**Volume of Right Rectangular Prisms**

In Topic C, students extend their understanding of the volume of a right rectangular prism with integer side lengths to right rectangular prisms with fractional side lengths. They apply the known volume formula, \( V = lwh \) and extend the formula to include the formula \( V = \text{area of base} \times \text{height} \). Students find the area of the base and multiply that number times the height.

**Volume with Fractional Edge Lengths and Unit Cubes**

This problem gives an excellent chance for students to review volume of a rectangular prism with whole numbers.

**Problems and Solutions**

1. Which prism will hold more 1 in x 1 in x 1 in cubes? How many more cubes will the prism hold?

   \[
   V = lwh \\
   V = 10 \text{ in} \times 4 \text{ in} \times 6 \text{ in} \\
   V = 240 \text{ in}^3 \\
   \]

   The prism with 480 in\(^3\) holds the most 1 in x 1 in x 1 in cubes. It holds 240 more cubes than the smaller prism.

2. Calculate the volume of this right rectangular prism when the side lengths are not whole numbers.

   \[
   V = \frac{1}{3} \text{ cm} \times \frac{2}{3} \text{ cm} \times \frac{1}{3} \text{ cm} \\
   V = \frac{16}{27} \text{ cm}^3 \text{ or } \frac{20}{27} \text{ cm}^3
   \]
Focus Area Topic C

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Problems and Solutions

1. A toy company is packaging its toys to be shipped. Some of the very small toys are each placed inside a cube-shaped box with side lengths of \( \frac{1}{2} \) in. These smaller boxes are then packed into a shipping box with dimensions of: 12 in x 4\( \frac{1}{2} \) in x 3\( \frac{1}{2} \) in

How many small toys can be packed into the larger box for shipping?

Because the sides of the small boxes containing the toys are \( \frac{1}{2} \) inch in length, each box will hold 2 small boxes for every inch in the larger box.

\[
\begin{align*}
12 \text{ in} &\times \frac{1}{2} \text{ in} \times \frac{1}{2} \text{ in} \\
24 \text{ in} &\times 9 \text{ in} \times 7 \text{ in} = 1512
\end{align*}
\]

1,512 small toys will fit in the larger box.

2. Write a numerical expression for the volume of each of the rectangular prism.

\[
\begin{align*}
(15 \text{ in})(1\frac{1}{2} \text{ in})(3 \text{ in}) &\quad (15 \text{ in})(1\frac{1}{2} \text{ in})(6 \text{ in}) \\
(15 \text{ in})(1\frac{1}{2} \text{ in})(9 \text{ in})
\end{align*}
\]

a. What do all of these expressions have in common? 
All of the expressions have \((15 \text{ in})(\frac{3}{2} \text{ in})\).

b. What do they represent?
This is the area of the base.

c. Rewrite the numerical expressions to show what they have in common.

\[
\begin{align*}
(15 \text{ in})(1\frac{1}{2} \text{ in})(3 \text{ in}) &\quad (15 \text{ in})(1\frac{1}{2} \text{ in})(6 \text{ in}) \\
(15 \text{ in})(1\frac{1}{2} \text{ in})(9 \text{ in}) \\
22\frac{1}{2} \text{ in}^3 &\quad (22\frac{1}{2} \text{ in}^3)(6 \text{ in}) \\
(22\frac{1}{2} \text{ in}^3)(9 \text{ in})
\end{align*}
\]

d. If we know volume for a rectangular prism as length times width times height, what is another formula for volume that could be used based on these examples?
We could use \( V = (\text{area of the base})(\text{height}) \) or \( V = Bh \) where the “B” stands for area of the base.

\[
\begin{align*}
V &= Bh \\
V &= (22\frac{1}{2} \text{ in}^3)(3 \text{ in}) \\
V &= 67\frac{1}{2} \text{ in}^3
\end{align*}
\]

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Problems and Solutions

3. Complete and use the table below to determine the relationship between the width and volume of this rectangular prism.

<table>
<thead>
<tr>
<th>Width in feet</th>
<th>Volume in cubic feet</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/10</td>
<td>2</td>
</tr>
<tr>
<td>6/10</td>
<td>4</td>
</tr>
<tr>
<td>12/10</td>
<td>8</td>
</tr>
<tr>
<td>24/10</td>
<td>?</td>
</tr>
</tbody>
</table>

a. What happened to the volume when the width was doubled? The volume doubled.

b. What happened to the volume when the width was quadrupled? The volume quadrupled.

c. What conclusion can you make when the base area stays constant and only the width changes?

Each time the width is multiplied by a number, the original volume will be multiplied by the same amount.

4. If \( B \) represents the area of the base and \( w \) represents the the width, write an expression that represents the volume.

\[ V = Bh \]

The area of the base of a rectangular fish tank is 25 \( \text{ in}^2 \). If the height of this fish tank is 18 inches, what is the volume of the fish tank in inches cubed?

\[ V = BH \]
\[ V = 25 \text{ in}^2 \times 18 \text{ in.} \]
\[ V = 450 \text{ in}^3 \]